SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2007

MATHEMATICS

8:45 am – 11:50 am
Tuesday 28th August 2007

Directions to Students

• Reading Time : 5 minutes	• Total Marks **120**
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• Attempt Question 1 – 10
• Board approved calculators may be used	• All questions are of equal value
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

Total marks (120)
Attempt Questions 1 – 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\pi^{-1.6}$ correct to 2 significant figures. **2**

(b) Write down the exact value of

(i) $\operatorname{cosec}\dfrac{\pi}{6}$ **1**

(ii) $10^{\log_{10}\sqrt{2}}$ **1**

(c) Paul paid Jonestown mechanical repairs \$510.00 for his car to be serviced. **2**
This price included a 15% discount from the standard cost of the service.
What was the standard cost of the service.

(d) Factorise fully $2-54x^3$. **2**

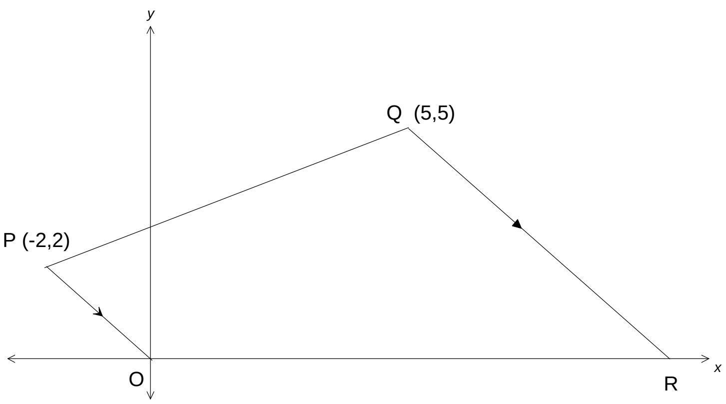
(e) Solve $|y-2|=6$, for y. **2**

(f) Solve $5-\dfrac{a}{9}<3$, for a. **2**

QUESTION 2 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a)

Not to Scale



In the diagram, OPQR is a trapezium with OA parallel to RQ. The co-ordinates of O, P and Q are (0, 0), (-2, 2) and (5, 5) respectively.

(i) Calculate the exact length of OP. **1**

(ii) Write down the gradient of OP. **1**

(iii) What is the size of angle POR. **1**

(iv) Find the equation of the line QR, and hence find the co-ordinates of R. **2**

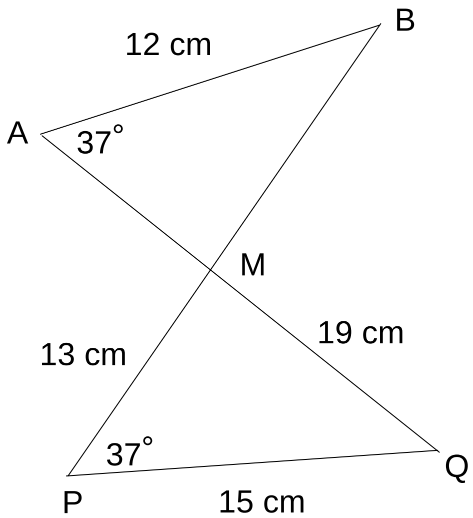
(v) Show that the perpendicular distance from O to RQ is $5\sqrt{2}$ units. **2**

(vi) Hence, or otherwise, calculate the area of the trapezium OPQR. **2**

(b) Shade the region in the Cartesian plane for which the inequalities $y < x-1$, $y \geq 0$ and $x \geq 4$ hold simultaneously. **3**

QUESTION 3 (12 MARKS) Use a SEPARATE writing booklet **Marks**

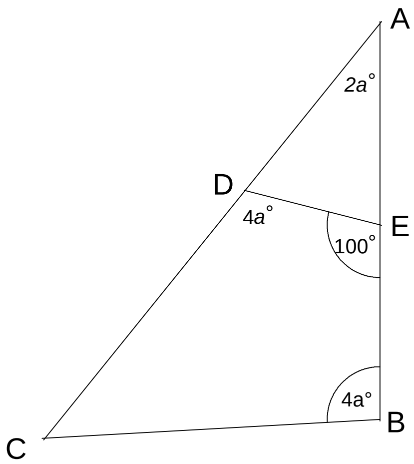
(a) **2**



Not to Scale

In the diagram, triangles BAM and QPM are similar. BA = 12 cm, QP = 15cm, QM = 19 cm and PM = 13 cm. Find the length of side AM.

(b) **3**

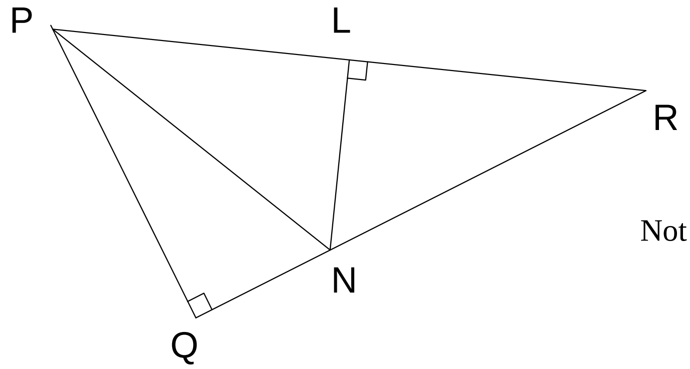


Not to Scale

In the diagram ABC is a triangle and E and D are points on AB and AC respectively. Angle CAB = $2a^\circ$, angle EBC = angle CDE = $4a^\circ$ and angle DEB = 100°.
Find the value of a showing full working and give reasons.

QUESTION 3 (Continued) **Marks**

(c)



Not to Scale

In the diagram PQR is a triangle with a right angle at Q. L is the midpoint of PR, and N is the point where the perpendicular to PR at L meets QR.

(i) Show that the triangles PNL and RNL are congruent, giving reasons. **2**

(ii) Suppose that it is also given that NP bisects the angle QPR. Find

(α) the size of angle NRL **2**

(β) the exact ratio LN : PR **2**

(d) Write down the sum of the interior angles of a regular Heptagon (7 sided polygon). **1**

QUESTION 4 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) Differentiate with respect to x :

(i) $y = \log_e\left(3 + 2x^2\right)$. **1**

(ii) $y = \left(3e^x - 5\right)^7$. **2**

(iii) $y = \dfrac{\sin 3x}{x}$ **2**

(iv) $y = x\sqrt{x}$ **2**

(b) Find the equation of the tangent to the curve $y = -\dfrac{2}{x}$ at the point (-1,2). **3**

(c) Find $\lim\limits_{x \to \infty} \left(\dfrac{x^2 - 3x + 1}{2x^2 + 5} \right)$ **2**

QUESTION 5 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) The roots of the equation $2x^2+4x-1=0$ are α and β.

(i) Write down the values of $(\alpha+\beta)$ and $\alpha\beta$. **1**

(ii) Evaluate $(\alpha^2+\beta^2)$ **2**

(b) Consider the quadratic equation $x^2-(k-2)x+(k+1)=0$,
where 'k' is a constant.

(i) Show that the discriminant is (k^2-8k). **2**

(ii) Find the values of 'k' for which the equation has real roots. **1**

(c) A parabola has an equation in the form $x^2-12x=8y-52$

(i) Express this equation in the form $(x-h)^2=4a(y-k)$ **1**

(ii) Hence find the:

(α) co-ordinates of the vertex **1**

(β) co-ordinates of the focus **1**

(γ) equation of the directrix **1**

(d) Solve the equation $(5^x)^2+5^x-2=0$, for x. **2**

QUESTION 6 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) Find

(i) $\int \frac{1}{4x^4}\, dx$ **1**

(ii) $\int \frac{2x}{x^2+10}\, dx$ **1**

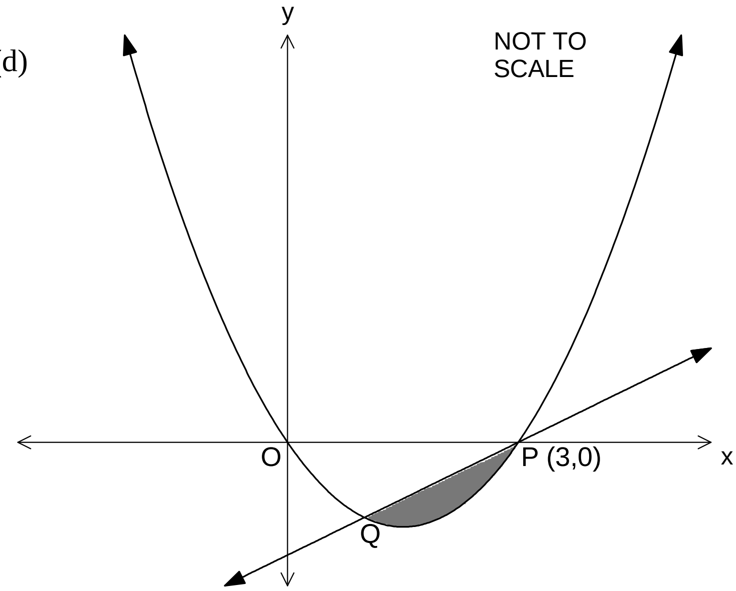
(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sec^2 x \, dx$ **2**

(c) Use Simpson's Rule with three function values to find an approximation for **3**

$\int_3^7 \frac{x}{\ln x}\, dx$.

Give your answer correct to one decimal place.

(d)



The graphs of $y = x - 3$ and $y = x^2 - 3x$ intersect at the points P (3,0) and Q, as shown in the diagram.

(i) Find the co-ordinates of Q. **2**

(ii) Find the area of the shaded region bounded by $y = x^2 - 3x$ and $y = x - 3$. **3**

QUESTION 7 (12 MARKS) Use a SEPARATE writing booklet **Marks**

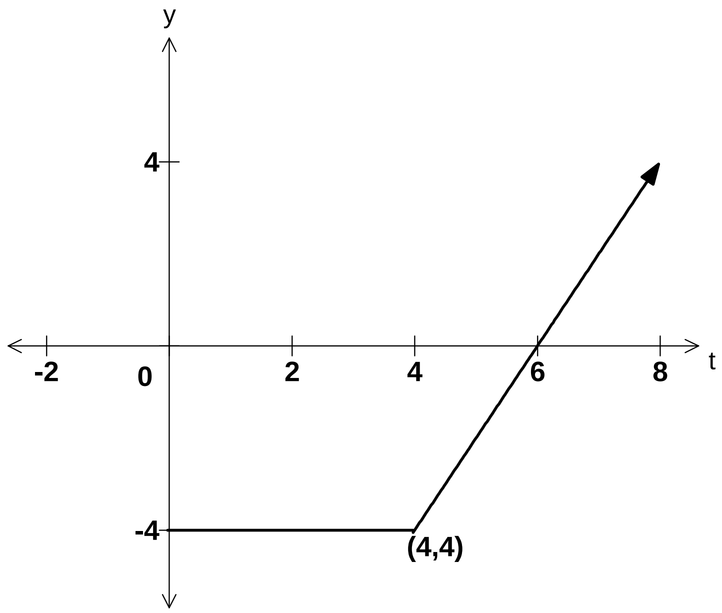
(a) Consider the curve $y = x^3 - 3x^2 + 3x - 1$.

(i) Show that the curve has only one stationary point, find its co-ordinates and determine its nature. **3**

(ii) State the values of x for which the curve is concave up. **1**

(iii) State the values of x for which the curve is increasing. **1**

(b) **2**



The graph shows the function $y = f(x)$ whose domain is $x \geq 0$.

Trace or copy this graph onto your answer booklet

On the same axes, sketch the graph of the function $y = f'(x)$.

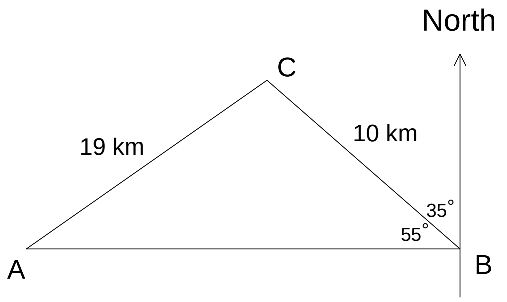
(c) The curve $y = f(x)$, $0 \leq x \leq 2\pi$ has a stationary point at $x = \frac{\pi}{4}$ and $f''(x) = \cos x + \sin x$. **3**

Find the x co-ordinate at which there is another stationary point.

(d) If $y = x\sqrt{x+1}$, show that $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$. **2**

QUESTION 8 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram, the point B is due east of point A. The point C is 19 km from point A and 10 km from point B. Point C is North 35° West of point B. Find the true bearing of point C from point A. **3**

(b) The velocity of a particle, moving in a straight line, is given by

$$\frac{dx}{dt} = 2 - 4\sin t \text{ for } 0 \le t \le 2\pi$$

where $\frac{dx}{dt}$ is measured in metres per second, t is measured in seconds and x in metres.

(i) At what times during this period is the particle at rest. **2**

(ii) Sketch the graph of velocity $\frac{dx}{dt}$ as a function of time for $0 \le t \le 2\pi$. **2**

(This sketch should be approximately half of one page)

(iii) What is the maximum velocity of the particle during this period? **2**

(iv) Calculate the exact total distance travelled by the particle between $t = 0$ and $t = \frac{\pi}{2}$ **3**

QUESTION 9 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) The population P of ants in a colony is determined by $P = 500 \times e^{\frac{t}{4}}$ where t is the time in weeks since the colony was originally established.

(i) Find the size of the colony after 10 weeks. **1**

(ii) How long would it take for the population to reach 10 000? **1**

(iii) Find $\frac{dP}{dt}$ and explain what this means for the population trend? **2**

(iv) Sketch the graph of $\frac{dP}{dt}$ against t. **2**

(b) Paul and Wendy borrow \$20000 from the Miami Bank. This loan plus interest is to be repaid in equal monthly instalments of \$399 over five years. Interest of 7.2% p.a is compounded monthly on the balance owing at the start of each month.

Let $\$A_n$ be the amount owing after n months.

(i) Over the five year repayment period, how much interest is charged? **1**

(ii) Show that $A_1 = 19721$ **1**

(iii) Clearly show that $A_2 = 20000 \times 1.006^2 - 399(1 + 1.006)$. **1**

(iv) Deduce then that $A_n = 66500 - 46500 \times 1.006^n$ **2**

(v) After two years of repayments Paul and Wendy decide on the very next day to repay the loan in one full payment. How much will this one payment be? **1**

QUESTION 10 (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) The area bounded by the curve $y=\frac{1}{x}$, the x-axis and the ordinates $x=a$ and $x=4$ is rotated about the x-axis. If the volume generated is $\frac{\pi}{2}$ unit^3, where $0<a<4$, find the value of a. **3**

(b) If $x\sec\theta = y\tan\theta$, prove that $\tan\theta\sec\theta = \frac{xy}{y^2-x^2}$ **3**

(c) If the straight line $y=mx$ is a tangent to the curve $y=e^{\frac{x}{2}}$, find the exact value of *m*. Clearly show your working. **3**

(d) The sum of the first 'n' terms of a series is given by $S_n = n^2 + 6n$. **Prove** that this series is arithmetic. **3**

(Note: It will be insufficient to outline a proof which involves evaluating S_1, S_2, *etc.*, T_1, T_2, *etc.*)

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Solutions

B.L.C

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2007

MATHEMATICS

- This is a study resource for HSC preparation.
- This document is designed to help students understand the questions.
- The solutions should be treated as aids only.
- There may be better solutions to some questions

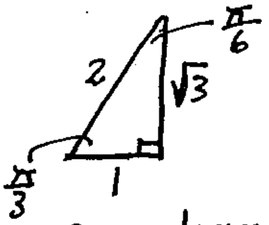
Question 1

(a) $\pi^{-1.6} \doteqdot 0.16$

From calculator
0.160162265...

(a) 1AW - Correct calc^c
1AW - Correct rounding

(b) (i) $\operatorname{cosec} \frac{\pi}{6} = \frac{2}{1} = 2.$



(i) 1AW - Correct answer

(ii) $10^{\log_{10}\sqrt{2}} = \sqrt{2}$ Rule: $a^{\log_a x} = x$

(ii) 1AW - Correct answer

(c) 85% of cost of standard service = \$510.00
1% of cost of standard service = $\$\frac{510.00}{85}$
cost of standard service = $\$\left(\frac{510.00 \times 100}{85}\right)$
= \$ 600.00

1AW - Some attempt at unitary method
1AW - Correct answer

(d) $2 - 54x^3 = 2(1-27x^3)$
$= 2(1-3x)(1+3x+9x^2)$

1AW - take out a factor of 2
1AW - correct answer

(e) $|y-2| = 6$
$\pm(y-2) = 6$
$y-2 = 6$ or $y-2 = -6$
$y = 8$ or $y = -4$

(e) Well done by most
1AW - $y = 8$
1AW - $y = -4$

(f) $5 - \frac{a}{9} < 3$
$-\frac{a}{9} < -2$
$a > 18$

(f) Many careless errors made here
1AW - Reasonable attempt
1AW - Correct answer

Question 2

(i) $O(0,0)$; $P(-2,2)$

$OP^2 = 2^2 + 2^2$
$OP^2 = 4 + 4$
$OP^2 = 8$
$OP = \sqrt{8} = 2\sqrt{2}$

(i) Well answered

(ii) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$m = \dfrac{2-0}{-2-0}$
$m = -1$

(ii) well answered

(iii) $\tan\theta = m$
$\tan \widehat{POR} = -1$
$\widehat{POR} = 135^\circ$

(iii) many students did not know this result !!

(iv) $QR \parallel PO$
$m_{QR} = m_{PO} = -1$
Equation of QR :
$y - y_1 = m(x - x_1)$
using $Q(5,5)$
$y - 5 = -1(x-5)$
$y - 5 = -x + 5$
$x + y - 10 = 0$
Now let $y=0$ for co-ordinates of R
$\therefore x = 10$
point $R = (10,0)$

(iv) many students did not read the second part of this question ----
→ Find the co-ord. of R

1AW - Equation of Line
1AW - Co-ord. of R

(v) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

BC : $x + y - 10 = 0$
Point O $(0,0)$

$d = \dfrac{|1\times 0 + 1\times 0 - 10|}{\sqrt{1^2+1^2}}$

$d = \dfrac{10}{\sqrt{2}}$

$d = \dfrac{10}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$d = 5\sqrt{2}$ units

(v) Generally well answered

1AW - Substitution into Formula

1AW - Rationalising the denominator.

(vi) Distance QR.

$\overline{QR}^2 = (5-10)^2 + (5-0)$

$\overline{QR}^2 = (-5)^2 + 5^2$

$\overline{QR}^2 = 25 + 25$

$\overline{QR}^2 = 50$

$\overline{OR} = \sqrt{50} = 5\sqrt{2}$

Trapezium : $A = \dfrac{h}{2}(a+b)$

$A = \dfrac{5\sqrt{2}}{2}\left(2\sqrt{2} + 5\sqrt{2}\right)$

$A = \dfrac{5\sqrt{2}}{2} \times 7\sqrt{2}$

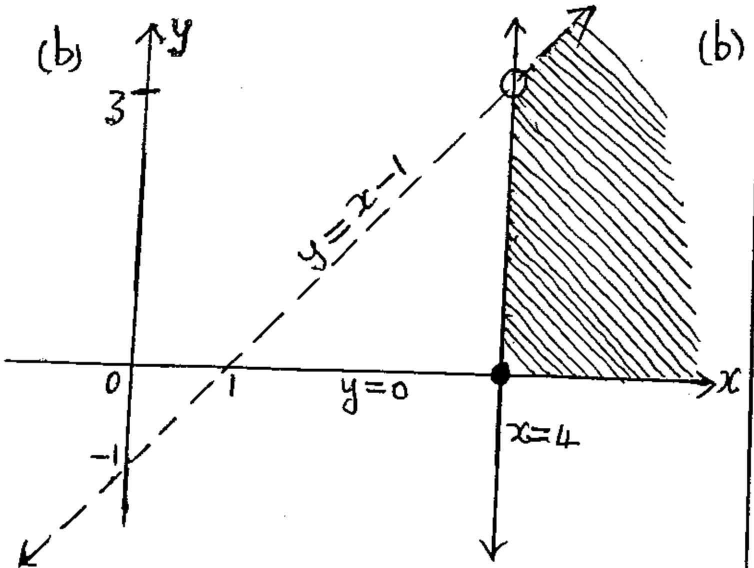
$A = 35$ units^2.

(vi) Many students had difficulty with this

1AW - Distance QR

1AW - Use of Trapezium formula.

(b)



(b) Poorly answered by many.

Marks were <u>deducted</u> for

1. Not showing intercepts
2. Not labling diagram
3. Not showing dotted line for $y = x - 1$

<u>Note</u> - use a ruler
- use a pencil.

Question 3

(a) $\frac{AM}{13} = \frac{12}{15}$ — 1 — Well done

$AM = \frac{12 \times 13}{15}$ — 1

$AM = 10 \cdot 4 \text{ cm}$

(Note: Ratios of corresponding sides are equal)

(b) $\widehat{AED} = 180° - 100° = 80°$ — 1

(Angle sum at E on a straight line AB is 180°)

$2a° + 80° = 6a°$. External angle of ΔADE equals the sum of the interior opposite angles. — 1

$\therefore 4a = 80$ — 1

$a = 20$

MANY DIFFERENT OPTIONS USED. QUITE WELL DONE.

$a = 40$ WITHOUT SOME CORRECT SETTING OUT WAS PENALISED

Note: There is more than one approach here.

(c)

(i) In the Δ's PNL and RNL

NL is a common side

$\widehat{NLP} = \widehat{NLR} = 90°$ ($NL \perp PR$ given)

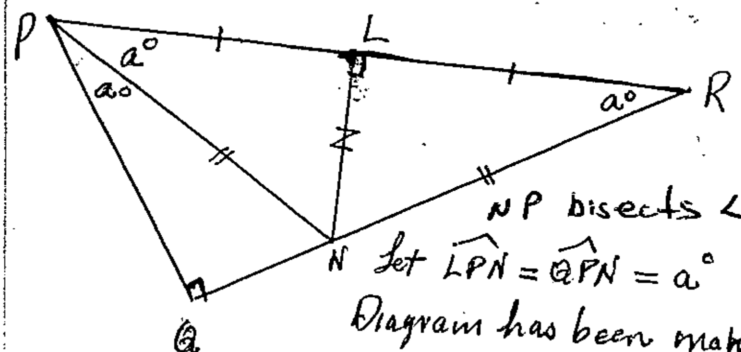
$LP = LR$ (L is the midpoint of PR, given) — 1

1 MK AWARDED FOR FINDING ALL THREE PARTS OF PROOF

$\therefore \Delta PNL \equiv \Delta RNL$ (SAS) — 1

FOR CORRECT CONGRUENCY RULE

(ii)



NP bisects $\angle QPN$. Let $\widehat{LPN} = \widehat{QPN} = a°$

Diagram has been marked with equality data.

MANY HAD NO REASON OFFERED FOR $\angle QPN = \angle NPL$.

(II) continued.

(α) In Δ PQR $2a^\circ + a^\circ + 90^\circ = 180^\circ$

$3a = 90$

$a = 30$

$\therefore \widehat{NRL} = 30^\circ$

(β) $\frac{LN}{PL} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

Now since $PL = \frac{1}{2} PR$.

We have: $\frac{LN}{\frac{1}{2}PR} = \frac{1}{\sqrt{3}}$

$\therefore \frac{LN}{PR} = \frac{1}{2\sqrt{3}}$

(d) Formula: $(2n-4)$ right-angles

$n = 7$

Angle sum $= (14-4) \times 90^\circ$

$= 900^\circ$.

1 AGAIN, A NUMBER OF CORRECT OPTIONS USED. MOST SUCCESSFULLY.

1

1 MANY HAD GOOD SOLN'S TO THIS. SOME MADE A SOLID START BUT COULD NOT COMPLETE. PART MKS AWARDED

1

I Well done.

Marks

Question 4

(a) (i) $y = \log_e(3+2x^2)$

$\frac{dy}{dx} = \frac{4x}{3+2x^2}$ 1 Many forgot formula.

(ii) $y = (3e^x - 5)^7$

$\frac{dy}{dx} = 7(3e^x-5)^6 \times 3e^x$ 1 many forgot to differentiate $3e^x$.

$= 21e^x(3e^x-5)^6$ 1

(iii) $y = \frac{\sin 3x}{x}$

$\frac{dy}{dx} = \frac{x \times 3\cos 3x - (\sin 3x) \times 1}{x^2}$ 1 for finding u' and v'

$= \frac{3x\cos 3x - \sin 3x}{x^2}$ 1 correct arrangement in formula.

(iv) $y = x\sqrt{x} = x \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$ 1 changing to index form ie $x^{1\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ 1 differentiating correctly

$= \frac{3\sqrt{x}}{2}$

many used product rule but this was the long way round.

(b) $y = -2x^{-1}$, point $(-1, 2)$

$\frac{dy}{dx} = 2x^{-2} = \frac{2}{x^2}$ 1 differentiating correctly

1 gradient of tangent

at $x=-1$, $m = \frac{2}{(-1)^2} = 2$

tangent: $y - y_1 = m(x - x_1)$

$y - 2 = 2(x+1)$

$y - 2 = 2x + 2$ 1 for equation of line

$2x - y + 4 = 0$

alternative: $y = 2x + 4$

(c) $\displaystyle \text{Limit}_{x \to \infty} \left(\frac{x^2 - 3x + 1}{2x^2 + 5} \right)$

$\displaystyle = \text{Limit}_{x \to \infty} \left(\frac{1 - \frac{3}{x} + \frac{1}{x^2}}{2 + \frac{5}{x^2}} \right)$

$\displaystyle = \frac{1 - 0 + 0}{2 + 0}$

$\displaystyle = \frac{1}{2}$

1 dividing through by the highest power of x ie x^2

$\displaystyle = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{2x^2}{x^2} + \frac{5}{x^2}}$

1 correct answer.

Question 5

(a) $2x^2+4x-1=0$

$a=2, b=4, c=-1$

(i) $\alpha+\beta = -\frac{b}{a} = -\frac{4}{2} = -2$

$\alpha\beta = \frac{c}{a} = -\frac{1}{2}$

Both right ✓

(ii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2(-\frac{1}{2})$ ✓

$= 4+1$

$= 5.$ ✓

(b) $x^2-(k-2)x+(k+1)=0$

(i) $\Delta = b^2-4ac$

$\Delta = [-(k-2)]^2 - 4(1)(k+1)$

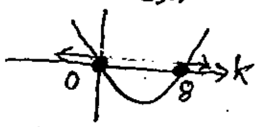
$\Delta = k^2-4k+4-4k-4.$ ✓

$\Delta = k^2-8k$ ✓

(ii) $\Delta \geq 0$ for real roots

$k^2-8k \geq 0$

$k(k-8) \geq 0$



Ans $k \leq 0$ or $k \geq 8$ ✓

Not many problems in this Q.

(c) $x^2 - 12x = 8y - 52$

$x^2 - 12x + (-6)^2 = 8y - 52 + 36$

$(x-6)^2 = 8y - 16$

$(x-6)^2 = 8(y-2)$

$(x-6)^2 = 4(2)(y-2)$ ✓

(d) Vertex : $(6, 2)$ ✓

Focal length = 2.

(β) Focus : $(6, 4)$ ✓

(γ) Directrix : $y = 0$ ✓

* If brackets ( ) were not used for coordinates 1 mark was deducted in (α) and (β)

(d) $(5^x)^2 + 5^x - 2 = 0$

Let $t = 5^x$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2$ or $t = 1$ ✓

$\therefore 5^x = -2$ N/A as $5^x > 0$ for all x

$5^x = 1$

$5^x = 5^0$

$x = 0$ ✓

Question 6

(a) (i) $\int \frac{1}{4x^4} dx$

$= \int \frac{1}{4} x^{-4} dx$

$= \frac{1}{4} \frac{x^{-3}}{-3} + c$

$= -\frac{1}{12x^3} + c$ ✓

Many incorrectly dealt with this question as a logarithm. Two issues to deal with...

(1) $\frac{1}{4}$

(2) x^{-4}

(ii) $\int \frac{2x}{x^2+10} dx$

$= \ln(x^2+10) + c$ ✓

answered well.

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sec^2 x \, dx$

$= \left[\tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ ✓

$\int_a^b \sec^2 x = \left[\tan x\right]_a^b$

$= \tan\frac{\pi}{4} - \tan\frac{\pi}{6}$

$= 1 - \frac{1}{\sqrt{3}}$ ✓

exact value leave in this form.

$= \frac{\sqrt{3}-1}{\sqrt{3}}$ or $\frac{\sqrt{3}(\sqrt{3}-1)}{3}$ or $\frac{3-\sqrt{3}}{3}$

(c) $\int_3^7 \frac{x}{\ln x} dx$

x	3	5	7
$\frac{x}{\ln x}$	$\frac{3}{\ln 3}$	$\frac{5}{\ln 5}$	$\frac{7}{\ln 7}$
	y_1	y_2	y_3

✓ completing the table of values

$I \doteqdot \frac{b-a}{h}\left[(y_1+y_3)+4y_2\right]$

$I \doteqdot \left(\frac{7-3}{6}\right)\left[(2\cdot7307+3\cdot5973)+4(3\cdot1067)\right]$

✓ correctly substituting into formula

NOTE: $\frac{h}{3} = \frac{2}{3}$

$I \doteqdot 12\cdot5$ c.1.d.p ✓

(d) (i) $y = x - 3 \quad --- ①$

$y = x^2 - 3x \quad --- ②$

$② - ①: x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1$ or 3.

when $x = 1$ $y = -2$

$\therefore$ point Q is $(1, -2)$ ✓

✓

(ii)

$$A = \int_1^3 (x^2 - 3x)\,dx - \int_1^3 (x-3)\,dx$$

$$A = \int_1^3 (x^2 - 4x + 3)\,dx \quad ✓$$

$$A = \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3 \quad ✓$$

$$A = \left[(9 - 18 + 9) - \left(\tfrac{1}{3} - 2 + 3\right)\right]$$

$$A = 1\tfrac{1}{3} \text{ units}^2 \quad ✓$$

Area between two curves is best answered using the formula: $\int_a^b f(x) - g(x)\,dx$.

Note: 1 mark deducted if you had the area as $-\frac{4}{3}$ units^2.

Question 7

(a) $y = x^3 - 3x^2 + 3x - 1$

$\frac{dy}{dx} = 3x^2 - 6x + 3$

$\frac{d^2y}{dx^2} = 6x - 6$

(i) Let $\frac{dy}{dx} = 0$

$3x^2 - 6x + 3 = 0$

$3(x^2 - 2x + 1) = 0$

$(x-1)^2 = 0$ ✓ finding the x-value of the S.P

$x = 1$

hence only one stationary point.

When $x = 1$ $y = 1 - 3 + 3 - 1 = 0$ ✓ y-value of S.P

$\frac{d^2y}{dx^2} = 6(1) - 6 = 0$

This will be a horizontal point of inflection at $(1,0)$ ✓ determining nature of S.P

Concavity Test:

x	0	1	2
$\frac{d^2y}{dx^2}$	-6	0	6

ie $-\ \frac{d^2y}{dx^2}\ +$ (0)

This confirms inflection.

← marks weren't awarded for the concavity test but this should be shown.

(ii) Concave up: $f''(x) > 0$

$6x - 6 > 0$

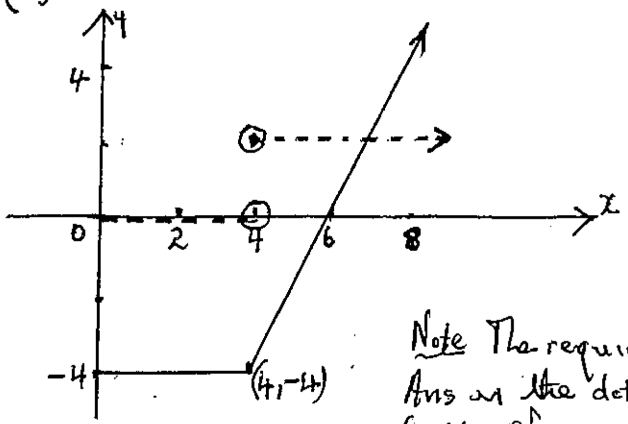
$x > 1$ ✓

(iii) $f'(x) > 0$

$(x-1)^2 > 0$

all real x, except $x \neq 1$. ✓

(b)



Note The required Ans is the dotted Graph. Of course unconventional.

✓ } lines $y=2$ ~~or~~ and $y=1$

✓ discontinuity at $x=4$

(c) $f''(x) = \cos x + \sin x$

$f'(x) = \sin x - \cos x + c$

Now when $f'(x) = 0$, $x = \frac{\pi}{4}$ $\therefore c = 0$

$\therefore f'(x) = \sin x - \cos x$ ✓

For stationary points $f'(x) = 0$

$\sin x - \cos x = 0$

$\sin x = \cos x$

$\tan x = 1 > 0$ ✓

In the stated domain

x in 1st and 3rd quadrant

Basic acute angle $= \frac{\pi}{4}$

Hence the other stationary point is at $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ ✓

(d) $y = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = 1 \times (x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}} \times 1 \times x$ ✓ correct application of product rule.

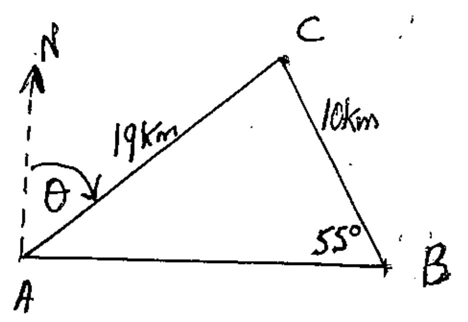
$= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

$= \frac{2(x+1)+x}{2\sqrt{x+1}}$ ✓ simplification.

$= \frac{3x+2}{2\sqrt{x+1}}$

Question 8

(a)



using The sine rule :

$$\frac{\sin B\hat{A}C}{10} = \frac{\sin 55^\circ}{19}$$ ✓

$$\sin B\hat{A}C = \frac{10\sin 55^\circ}{19}$$

$$B\hat{A}C = 25^\circ 32'$$ ✓

True bearing of C from A :

$$\theta = 90^\circ - 25^\circ 32' = 64^\circ 28'$$

ie $T\,064^\circ 28'$ ✓

True bearing is written in 3 digits. Many missed out on that (Did not penalise this time)

(b) $v = \frac{dx}{dt} = 2 - 4\sin t$.

(i) The particle is at rest when $v = 0$

$2 - 4\sin t = 0$

$\sin t = \frac{1}{2}$ ✓

Now considering the domain $0 \le t \le 2\pi$

$\sin t > 0$ ∴ t in 1st, 2nd quads

Basic acute angle $= \frac{\pi}{6}$

∴ Particle at rest when $t = \frac{\pi}{6}$ sec

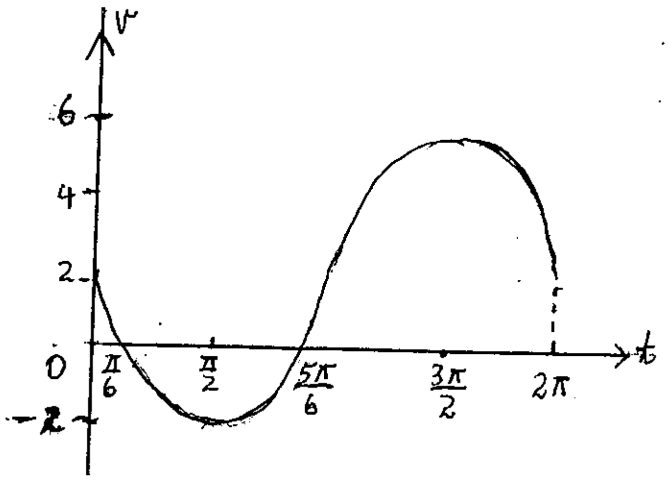
and when $t = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ sec ✓

Both $\frac{\pi}{6}$, $\frac{5\pi}{6}$ had to be there for 1 mark.

1 right No mark

more than 2 answers no mark.

(ii)



Points of intersection have to be shown on the t-axis ie $\frac{\pi}{6}$, $\frac{5\pi}{6}$

(iii) Maximum velocity occurs when sin t is a minimum
obviously $-1 \le \sin t \le 1$
This occurs at $t = \frac{3\pi}{2}$ ie $\sin t = -1$
$\therefore$ Max Velocity $= 2 - 4(-1)$
$= 6$ m/sec

Answer from graph accepted for 2 marks.

(iv) Now between $t = 0$ and $t = \frac{\pi}{2}$
The particle changes direction at $t = \frac{\pi}{6}$ (at $v = 0$)

$\therefore$ Total distance travelled

$$= \int_0^{\frac{\pi}{6}} (2 - 4\sin t)\,dt + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 - 4\sin t)\,dt$$

$$= \left| \left[2t + 4\cos t\right]_0^{\frac{\pi}{6}} \right| + \left[2t + 4\cos t\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left| \left(\frac{\pi}{3} + 4\cos\frac{\pi}{6}\right) - (0 + 4\cos 0) \right| + \left[\left(\pi + 4\cos\frac{\pi}{2}\right) - \left(\frac{\pi}{3} + 4\cos\frac{\pi}{6}\right)\right]$$

$$= \left| \frac{\pi}{3} + 4 \times \frac{\sqrt{3}}{2} - 4 \right| + \left[\pi + 4(0) - \frac{\pi}{3} - 4 \times \frac{\sqrt{3}}{2}\right]$$

$$= \left| \frac{\pi}{3} + 2\sqrt{3} - 4 \right| + \left| \pi - \frac{\pi}{3} - 2\sqrt{3} \right|$$

$$= \frac{\pi}{3} + 2\sqrt{3} - 4 + \left| + \frac{2\pi}{3} - 2\sqrt{3} \right|$$

$$= \frac{\pi}{3} + 2\sqrt{3} - 4 + \frac{2\pi}{3} + 2\sqrt{3} = \left(\frac{\pi}{3} + 4\sqrt{3} - 4\right)\text{m}$$

Very few reached the appropriate end. Anything close to 3rd step was awarded a mark.

Question 9

(a) $P = 500 \times e^{\frac{t}{4}}$

(i) Let $t = 10$

$P = 500 \times e^{\frac{10}{4}}$

$P = 500 \times e^{\frac{5}{2}}$

$P \doteqdot 6091$

generally well done.

1 correct answer

$\therefore$ After 10 weeks colony has 6091 ants

(ii) $t = ?$ when $P = 10000$

$500 \times e^{\frac{t}{4}} = 10000$

$e^{\frac{t}{4}} = 20$

$\frac{t}{4} = \ln 20$

$t = 4 \ln 20$

$t \doteqdot 11{\cdot}98$

generally well done.

1 correct answer

$\therefore$ Colony would take 11·98 weeks to reach 10,000.

(iii) $\frac{dP}{dt} = \frac{1}{4} \times 500 \times e^{\frac{t}{4}}$

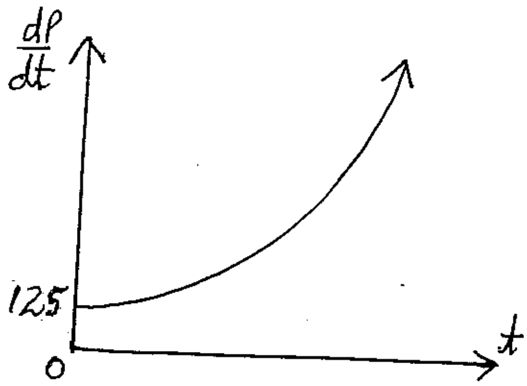
$= 125 \times e^{\frac{t}{4}}$

1 finding $\frac{dP}{dt}$

As $\frac{dP}{dt} > 0$ for all t, This means That The population will always be increasing.

1 explaining that population is increasing

(iv)



2 correct shape, vertical intercept + domain.

(b) Loan Amount = \$20000
Monthly Instalment \$399 over 5yrs
Interest 7.2% p.a compounded monthly.

(I) Interest $= (\$399) \times (12\times5) - \$20000$
$= \$3940$ | 1 | Simplest part of Q9 worst done.

(II) Using $A = PR^n$ where $R = 1 + \frac{r}{100}$

$A_1 = 20000(1.006)^1 - 399 = \$19721$ | 1 | had to show $20\,000 \times 1.006 - 399$

(III) $A_2 = A_1(1.006)^1 - 399$

$A_2 = [20000(1.006) - 399]1.006 - 399$ | 1 | had to show this step or equivalent.

$A_2 = 20000(1.006)^2 - 399(1 + 1.006)$

(IV) Continuing from part (III) and following the pattern apparent; Then after 'n' payments the amount A_n owing is:

$A_n = 20000(1.006)^n - 399(1 + 1.006 + \dots + 1.006^{n-1})$ | 1 | showing this expression for A_n

This is a GP with n terms, $a = 1$, $r = 1.006$

and using $S_n = \frac{a(r^n - 1)}{r-1}$ | many people faked from here to answer.

$\therefore A_n = 20000(1.006)^n - 399\left[\frac{1(1.006^n - 1)}{1.006 - 1}\right]$

$A_n = 20000(1.006)^n - 66500(1.006^n - 1)$ | 1 | must clearly show arithmetic.

$A_n = 20000(1.006)^n - 66500(1.006^n) - 66500$

$A_n = 66500 - 46500(1.006)^n$

(V) The One payment $= A_{24}$. | 1. | many forgot that 24 months = 2 yrs. Simple substitution into formula was given.

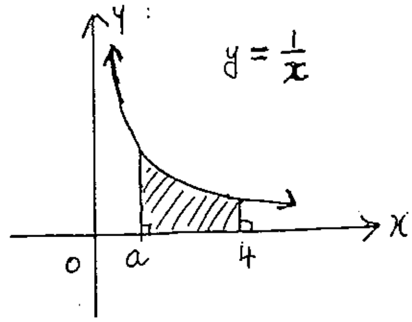
$A_{24} = 66500 - 46500(1.006)^{24}$

$A_{24} = \$12820.99$

or \$12821.

Question 10

(a)



$$V=\pi\int_a^b y^2\,dx$$

$$\frac{\pi}{2}=\pi\int_a^4 \frac{1}{x^2}\,dx$$

1 For correct integral.

$$\frac{1}{2}=\int_a^4 x^{-2}\,dx$$

Too many tried to use logs.

$$\frac{1}{2}=\left[\frac{x^{-1}}{-1}\right]_a^4$$

$$\frac{1}{2}=-\left[\frac{1}{x}\right]_a^4$$

$$\frac{1}{2}=-\left[\frac{1}{4}-\frac{1}{a}\right]$$

1 For correct substitution

$$\frac{1}{2}=\frac{1}{a}-\frac{1}{4}$$

$$\frac{1}{a}=\frac{3}{4}$$

$$a=\frac{4}{3}=1\tfrac{1}{3}.$$

1 For correct answer if all else correct

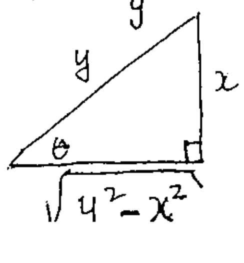
(b) $x\sec\theta = y\tan\theta$

$$\frac{x}{\cos\theta}=\frac{y\sin\theta}{\cos\theta}$$

$$x=y\sin\theta$$

$$\sin\theta=\frac{x}{y}$$

1 For correct discovery of $x=y\sin\theta$



1 For discovery of $\triangle$.

Now Prove $\tan\theta\sec\theta = \frac{xy}{y^2-x^2}$
using data from The triangle

LHS $= \tan\theta\sec\theta$

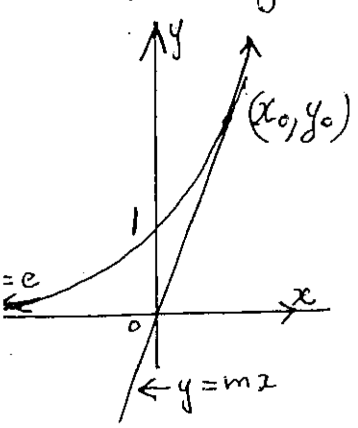
$= \frac{x}{\sqrt{y^2-x^2}} \times \frac{y}{\sqrt{y^2-x^2}}$

$= \frac{xy}{\sqrt{y^2-x^2}}$

$=$ RHS.

Many methods tried. Credit given for CORRECT substitution.
Too many assumed $\tan\theta\sec\theta = \frac{xy}{y^2-x^2}$.
1 mk for correct subs & evaluation.

(c) $y = mx$, $y = e^{\frac{x}{2}}$



Let (x_0, y_0) be The point of tangency.

$y = e^{\frac{x}{2}}$

$y' = \frac{1}{2}e^{\frac{x}{2}}$

$\therefore m = \frac{1}{2}e^{\frac{x_0}{2}}$ --- ①

Solving $y = mx$ and $y = e^{\frac{x}{2}}$

$mx = e^{\frac{x}{2}}$

So $mx_0 = e^{\frac{x_0}{2}}$.

Hence $\frac{1}{2}e^{\frac{x_0}{2}} \times x_0 = e^{\frac{x_0}{2}}$ from ①

$\frac{1}{2}x_0 = 1$

$x_0 = 2$

$\therefore m = \frac{1}{2}e^{2\times\frac{1}{2}}$

$m = \frac{e}{2}$.

1 For correct derivative
1 For attempt to solve two equations simultaneously
1 Correct evaluation of M.

(d) $S_n = n^2 + 6n$

$T_n = S_n - S_{n-1}$

$= n^2+6n-[(n-1)^2+6(n-1)]$

$= n^2+6n-[n^2-2n+1+6n-6]$

$= n^2+6n-n^2+2n-1-6n+6$

$= 2n+5$

$T_n - T_{n-1} = 2n+5-[2(n-1)+5]$

$= 2n+5-[2n-2+5]$

$T_n - T_{n-1} = 2n+5-2n+2-5$

$T_n - T_{n-1} = 2$ thus 'any term' minus the term before it is constant
Hence an AP.

1 $T_n = S_n - S_{n-1}$
Most assumed an AP excisted.
1 $T_n = 2n+5$
Many tried to find $T_1 T_2 T_3$ etc
1 $T_n - T_{n-1} = d$
$d = 2$.